

Research Council 2012a) and mathematics education (Donovan and Bransford 2005; Lester 2007) supports the characterization of mathematics learning as an active process, in which each student builds his or her own mathematical knowledge from personal experiences, coupled with feedback from peers, teachers and other adults, and themselves. This research has identified a number of principles of learning that provide the foundation for effective mathematics teaching. Specifically, learners should have experiences that enable them to—

- engage with challenging tasks that involve active meaning making and support meaningful learning;
- connect new learning with prior knowledge and informal reasoning and, in the process, address preconceptions and misconceptions;
- acquire conceptual knowledge as well as procedural knowledge, so that they can meaningfully organize their knowledge, acquire new knowledge, and transfer and apply knowledge to new situations;
- construct knowledge socially, through discourse, activity, and interaction related to meaningful problems;
- receive descriptive and timely feedback so that they can reflect on and revise their work, thinking, and understandings; and
- develop metacognitive awareness of themselves as learners, thinkers, and problem solvers, and learn to monitor their learning and performance.

Mathematics Teaching Practices

Eight Mathematics Teaching Practices provide a framework for strengthening the teaching and learning of mathematics. This research-informed framework of teaching and learning reflects the learning principles listed above, as well as other knowledge of mathematics teaching that has accumulated over the last two decades. The list on the following page identifies these eight Mathematics Teaching Practices, which represent a core set of high-leverage practices and essential teaching skills necessary to promote deep learning of mathematics.

Obstacles

Dominant cultural beliefs about the teaching and learning of mathematics continue to be obstacles to consistent implementation of effective teaching and learning in mathematics classrooms (Handal 2003; Philipp 2007). Many parents and educators believe that students should be taught as they were taught, through memorizing facts, formulas, and procedures and then practicing skills over and over again (e.g., Sam and Ernest 2000). This view perpetuates the traditional lesson paradigm that features review, demonstration, and practice and is still pervasive in many classrooms (Baniower et al. 2006; Weiss and Pasley 2004). Teachers, as well

Mathematics Teaching Practices	
Establish mathematics goals to focus learning.	Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving.	Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations.	Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse.	Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions.	Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding.	Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics.	Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking.	Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

as parents, are often not convinced that straying from these established beliefs and practices will be more effective for student learning (Barkatsas and Malone 2005; Wilken 2008).

In sharp contrast to this view is the belief that mathematics lessons should be centered on engaging students in solving and discussing tasks that promote reasoning and problem solving (NCTM 2009; National Research Council 2012a). Teachers who hold this belief plan lessons to prompt student interactions and discourse, with the goal of helping students make sense of mathematical concepts and procedures. However, the lack of agreement about what constitutes effective mathematics teaching constrains schools and school systems from establishing coherent expectations for high-quality, productive teaching of mathematics (Ball and Forzani 2011).

Teachers' beliefs influence the decisions that they make about the manner in which they teach mathematics, as indicated in the table at the right. Students' beliefs influence their

Beliefs about teaching and learning mathematics	
Unproductive beliefs	Productive beliefs
Mathematics learning should focus on practicing procedures and memorizing basic number combinations.	Mathematics learning should focus on developing understanding of concepts and procedures through problem solving, reasoning, and discourse.
Students need only to learn and use the same standard computational algorithms and the same prescribed methods to solve algebraic problems.	All students need to have a range of strategies and approaches from which to choose in solving problems, including, but not limited to, general methods, standard algorithms, and procedures.
Students can learn to apply mathematics only after they have mastered the basic skills.	Students can learn mathematics through exploring and solving contextual and mathematical problems.
The role of the teacher is to tell students exactly what definitions, formulas, and rules they should know and demonstrate how to use this information to solve mathematics problems.	The role of the teacher is to engage students in tasks that promote reasoning and problem solving and facilitate discourse that moves students toward shared understanding of mathematics.
The role of the student is to memorize information that is presented and then use it to solve routine problems on homework, quizzes, and tests.	The role of the student is to be actively involved in making sense of mathematics tasks by using varied strategies and representations, justifying solutions, making connections to prior knowledge or familiar contexts and experiences, and considering the reasoning of others.
An effective teacher makes the mathematics easy for students by guiding them step by step through problem solving to ensure that they are not frustrated or confused.	An effective teacher provides students with appropriate challenge, encourages perseverance in solving problems, and supports productive struggle in learning mathematics.

perception of what it means to learn mathematics and their dispositions toward the subject. As the table summarizes, the impact of these beliefs on the teaching and learning of mathematics may be unproductive or productive. It is important to note that these beliefs should not be viewed as good or bad. Instead, beliefs should be understood as unproductive when they hinder the implementation of effective instructional practice or limit student access to important mathematics content and practices.

Overcoming the obstacles

Teaching mathematics requires specialized expertise and professional knowledge that includes not only knowing mathematics but knowing it in ways that make it useful for the work of teaching (Ball and Forzani 2010; Ball, Thames, and Phelps 2008). Mathematics teaching

demands subject-specific understanding and insight so that teachers can skillfully carry out their work in mathematics classrooms. Some of the work of mathematics teaching includes finding an example or task to make a specific mathematical point, linking mathematical representations to underlying ideas and other representations, and evaluating students' mathematical reasoning and explanations. This work also requires teachers to be able to unpack mathematical topics that they know well and to reexamine these through the eyes of learners, as well as to be able to work with many learners simultaneously in classrooms, each with unique backgrounds, interests, and learning needs.

The following discussion and illustrations of the eight Mathematics Teaching Practices support the incorporation of the productive beliefs identified above into the daily professional work of effective teachers of mathematics. This framework offers educators within schools and across districts a common lens for collectively moving toward improved instructional practice and for supporting one another in becoming skilled at teaching in ways that matter for ensuring successful mathematics learning for all students.

Establish Mathematics Goals to Focus Learning

Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.

Effective mathematics teaching begins with a shared understanding among teachers of the mathematics that students are learning and how this mathematics develops along learning progressions. This shared understanding includes clarifying the broader mathematical goals that guide planning on a unit-by-unit basis, as well as the more targeted mathematics goals that guide instructional decisions on a lesson-by-lesson basis. The establishment of clear goals not only guides teachers' decision making during a lesson but also focuses students' attention on monitoring their own progress toward the intended learning outcomes.

Discussion

Mathematics goals indicate what mathematics students are to learn and understand as a result of instruction (William 2011). In fact, "formulating clear, explicit learning goals sets the stage for everything else" (Hiebert et al. 2007, p. 57). Goals should describe what mathematical concepts, ideas, or methods students will understand more deeply as a result of instruction and identify the mathematical practices that students are learning to use more proficiently. Teachers need to be clear about how the learning goals relate to and build toward rigorous standards, such as the Common Core State Standards for Mathematics. The goals that guide instruction, however, should not be just a reiteration of a standard statement or cluster but